Multifractal subgrid-scale modeling for large-eddy simulation. I. Model development and *a priori* testing

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Results are presented from a new approach to modeling the subgrid-scale stresses in large-eddy simulation of turbulent flows, based on explicit evaluation of the subgrid velocity components from a multifractal representation of the subgrid vorticity field. The approach is motivated by prior studies showing that the enstrophy field exhibits multifractal scale-similarity on inertial-range scales in high Reynolds number turbulence. A scale-invariant multiplicative cascade thus gives the spatial distribution of subgrid vorticity magnitudes within each resolved-scale cell, and an additive cascade gives the progressively isotropic decorrelation of subgrid vorticity orientations from the resolved scale Δ to the viscous scale λ_ν. The subgrid velocities are then obtained from Biot–Savart integrals over this subgrid vorticity field. The resulting subgrid velocity components become simple algebraic expressions in terms of resolved-scale quantities, which then allow explicit evaluation of the subgrid stresses τ_{ij}^*. This new multifractal subgrid-scale model is shown in *a priori* tests to give good agreement for the filtered subgrid velocities, the subgrid stress components, and the subgrid energy production at both low ($Re_\Delta \approx 160$) and high ($Re_\Delta \approx 2550$) resolved-scale Reynolds numbers. Implementing the model is no more computationally burdensome than traditional eddy-viscosity models. Moreover, evaluation of the subgrid stresses requires no explicit differentiation of the resolved velocity field and is therefore comparatively unaffected by discretization errors. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1965058]

I. INTRODUCTION

Large-eddy simulation (LES) in principle allows for significantly improved accuracy in simulating turbulent flows, by calculating the large-scale features of the flow while modeling the small scales. Since the large scales are unique to each flow, representing them with any universal turbulence model, as in traditional Reynolds-averaged modeling, is inherently problematic. The small scales of turbulent flows, on the other hand, become increasingly universal with progressively decreasing scale size, and are significant insofar as the flow itself is concerned only for their cumulative effect on the evolution of the large scales. Interactions between the large (resolved) scales and the small (unresolved) scales are accounted for by a subgrid-scale model.

Despite this promising framework, however, LES has yet to fulfill the role which its original proponents had intended.[1] In the 40 years since the development of the first practical subgrid-scale model by Smagorinsky,[2] LES approaches and subgrid-scale models have been proposed with some frequency, yet none has achieved the accuracy necessary for LES to become the preferred turbulence modeling method for the practicing engineer and scientist. While the past 15 years have seen modest improvements in LES techniques as well as application of LES to more complex flow regimes,[3] much of this apparent progress has resulted from the use of increasingly powerful computers, rather than from improvements to the underlying models and numerical methods. Since computer power will not increase fast enough to permit practical direct simulation of most turbulent flows for the foreseeable future,[4] further near-term advances in turbulent flow simulation must necessarily require improvements in turbulence modeling methods. Thus, development of accurate yet computationally efficient subgrid-scale models and related numerical techniques remain one of the central problems that must be solved to make LES a reliably accurate tool for a wide range of practical turbulent flow problems.

A. The subgrid stress tensor τ_{ij}

Large-eddy simulations commonly solve a form of the filtered momentum equation which, under incompressibility and commutivity of the filtering and derivative operators, can be written as

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\bar{u}_i\bar{u}_j + \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} - \nu\frac{\partial^2 \bar{u}_i}{\partial x_j} = -\frac{\partial}{\partial x_j}\tau_{ij}, \tag{1}$$

with the subgrid stress tensor τ_{ij} defined as

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