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An algebraic variational multiscale-multigrid-multifractal method (AVM^4) for large-eddy simulation of turbulent variable–density flow at low Mach number

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SUMMARY

An algebraic variational multiscale-multigrid-multifractal method is proposed for large-eddy simulation of turbulent variable-density flow at low Mach number. In the multifractal subgrid-scale modeling approach, the subgrid-scale quantities are explicitly evaluated from a multifractal description of associated gradient fields. The multifractal subgrid-scale modeling approach is embedded into a residual-based form of the variational multiscale method. A particular feature of the proposed form of the multifractal subgrid-scale modeling approach is scale separation by level-transfer operators from plain aggregation algebraic multigrid methods to identify the required smaller resolved scales. In this study, we introduce a novel development of the multifractal subgrid-scale modeling approach for application to turbulent variable–density flow at low Mach number. Based on the physical background, we derive a variable-density extension of the multifractal subgrid-scale modeling approach to recover the subgrid-scale velocity and temperature field. The proposed method is validated via two numerical test cases. First, turbulent flow in a channel with a heated and a cooled wall is considered for two different temperature ratios. Second, turbulent flow over a backward-facing step with heating is investigated. The results obtained with the algebraic variational multiscale-multigrid-multifractal method are compared with results obtained with the widely-used dynamic Smagorinsky model and a residual-based variational multiscale method. Particularly, the results obtained for turbulent flow in a channel with a heated and a cooled wall indicate the excellent prediction quality achievable by the proposed method for turbulent variable-density flow at low Mach number.

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KEY WORDS: large-eddy simulation; turbulent variable-density flow; multifractal subgrid-scale modeling; algebraic-multigrid scale separation; turbulent channel flow; backward-facing step

1. INTRODUCTION

Large-Eddy Simulation (LES) is a powerful approach to computing complex turbulent flows. Compared with Direct Numerical Simulation (DNS), which aims at numerically resolving all flow scales and, hence, requires disproportional computational power, LES consists of resolving the larger flow structures while modeling the effect of the more universal smaller flow structures on the larger flow structures. LES models for incompressible flow are discussed in an exhaustive form, for example, in [1] and for compressible flow, for example, in [2]. The low-Mach-number limit of the compressible Navier–Stokes equations may be used to describe a variety of turbulent variable-density flows, which cover a wide range of challenging applications

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