Multifractal subgrid-scale modeling for large-eddy simulation. II. Backscatter limiting and *a posteriori* evaluation

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Results are presented from *a posteriori* evaluations of momentum and energy transfer between the resolved and subgrid scales when the multifractal subgrid-scale model from Part I is implemented in a flow solver for large-eddy simulations of turbulent flows. The multifractal subgrid-stress model is used to evaluate the subgrid part τ_{ij}^* of the stress tensor, with the resolved part $\overline{\bar{u}_i\bar{u}_j}$ evaluated by an explicit filter. It is shown that the corresponding subgrid and resolved contributions $\mathcal{P}^*$ and $\mathcal{P}^R$ to the resolved-scale energetics produce extremely accurate results for the combined subgrid energy production field $\mathcal{P}(\mathbf{x},t)$. A separate backscatter limiter is developed here that removes spurious energy introduced in the resolved scales by including physical backscatter, without sacrificing the high fidelity in the stress and energy production fields produced by the multifractal subgrid-scale model. This limiter makes small reductions only to those components of the stress that contribute to backscatter, and principally in locations where the gradients are large and thus the energy introduced by numerical errors is also largest. Control of the energy introduced by numerical error is thus accomplished in a manner that leaves the modeling of the subgrid-scale turbulence largely unchanged. The multifractal subgrid-scale model and the backscatter limiter are then implemented in a flow solver and shown to provide stable and accurate results in *a posteriori* tests based on large-eddy simulations of forced homogeneous isotropic turbulence at cell Reynolds numbers ranging from $160 \leq Re_\Delta \leq 10^6$, as well as in simulations of decaying turbulence where the model and the limiter must adjust to the changing subgrid conditions.
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I. INTRODUCTION

A companion paper,[1] herein referred to as Part I, presented a new approach to modeling the subgrid stresses for large-eddy simulation (LES) of turbulent flows, based on the multifractal structure of the subgrid enstrophy field at inertial-range scales. That work outlined a method for modeling the subgrid vorticity field $\boldsymbol{\omega}^{sgs}(\mathbf{x},t)$ with a multifractal representation, and from this derived expressions for the subgrid velocity components $u_i^{sgs}(\mathbf{x},t)$ and the associated subgrid stress component fields $\tau_{ij}^*(\mathbf{x},t)$. Part I also presented results from *a priori* tests in which the multifractal subgrid-scale model was compared against direct numerical simulation (DNS) data for homogeneous isotropic turbulence. Those results showed that the model recovered the filtered subgrid velocity fields $\overline{u_i^{sgs}}$, the subgrid stresses τ_{ij}^*, and the subgrid energy production field $\mathcal{P}^*$ with significantly higher fidelity than is typically reported for models based on traditional eddy-viscosity or mixed scale-similarity approaches.

However, when any subgrid-stress model is implemented in an LES flow solver, then the results will reflect not only the fidelity of the subgrid-stress model, but also the effects of purely numerical errors introduced by the flow solver itself. These numerical errors include discretization and truncation errors introduced by discrete representations in the solver, aliasing errors introduced by nonlinear terms due to the finite resolution of the computations, and filtering and commutation errors introduced by implicit or explicit filters used in the solver. The aliasing errors, which arise from the inherent under-resolution in LES, inject spurious energy into the resolved scales of the flow and thereby act to destabilize the simulation. This spurious energy transfer is in addition to the natural energy exchange between the resolved and subgrid scales from physical interactions between the stress and strain-rate fields in the turbulent flow. Thus even with an "ideal" subgrid-scale turbulence model, namely one that in the absence of numerical errors always produces the exact subgrid-stress field $\tau_{ij}(\mathbf{x},t)$ when provided with exact resolved velocity field data, the flow solver must also include a means to remove the additional energy to remain stable and obtain physically realistic results.

Against this background, subgrid-scale models have often been treated as much as a means of stabilizing computations by removing the additional energy as they are a means

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